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EFFECTS OF VISCOSITY IN MAGNETOPLASMADYNAMIC GENERATORS

by

J.C. WU

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Joint Nuclear Research Center
Ispra Establishment - Italy

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Effects of viscosity in m.p.d. generators

J. C. Wu

List of Symbols

- $\langle\ \rangle$ = Averaged quantities, e.g. $\langle J\rangle = \dfrac{1}{2a}\displaystyle\int_{-a}^{a} J\,dy$
- D = Joule dissipation per unit volume of the generator channel.
- P = Delivered power per unit volume.
- V = Viscous dissipation per unit volume.
- W = Work done per unit volume by the fluid in pushing itself through the channel.
- R_i = $b/\sigma la$ = Generator internal resistance.
- R = External resistance, including the load resistance and other resistances (other than R_i).
- U_{max} = Maximum velocity of the fluid at the centre of the channel.
- m = $\sigma B^2/\rho u_0$ = Magnetic interaction parameter.
- β = $\sqrt{(\sigma/M)}Ba$ = Hartmann number.
- ϵ = Displacement thickness a.

Introduction

In studies of the efficiency and performance of magneto-plasmadynamic generators, one-dimensional or quasi-one-dimensional flow theories are frequently used and the plasmas are treated as inviscid fluids. While it is well recognized that aerodynamic or viscous losses do exist in practice, their effects are believed by many to be small and relatively unimportant[1]. Blackman *et al.*[2] however, suggested that the effect of viscosity is anything but small, estimating a reduction of delivered power by a factor of 4 due to the presence of boundary layers with displacement thicknesses one-tenth of the channel width. Way[3] reported discrepancies between experimentally-obtained and theoretically-predicted powers and pointed out that part of the discrepancies can probably be accounted for by 'leakage currents' associated with the presence of boundary layers. In the following paragraphs, some effects associated with the non-zero viscosity of the plasma in a cross-field d.c. generator are treated, and attempts are made to assess quantitatively the reduction in obtainable power densities.

Viscous Effects Associated with the Insulators

To study the effects associated with insulator surfaces, the assumption is made that all physical quantities are independent of z (see Fig. 1). The physical configuration approximated by

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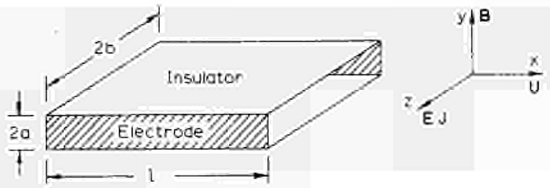

Fig. 1
Generator channel

this assumption is one in which b is much greater than a. It should be noted that a moderately large b/a is considered to be practical in the generator configuration for reasons of economical magnet design and from consideration of the electrode voltage drops. The case of constant μ and σ is considered. The constant-velocity (at the centre of the generator channel) configuration is selected for the present analyses on account of the simplicity it affords. The advantages of selecting such a configuration for actual designs have been reported by previous authors[4]. The velocity profile at some station, say x_0, of the generator channel is represented by $u = u_0 f(y)$, where u_0 is a reference velocity. For the analyses concerning the insulators, the most convenient reference velocity is related to $\partial p/\partial x$, σ, E, and B in the same manner as that of the inviscid velocity in a constant-area channel, i.e.

$$u_0 = \frac{-\partial p/\partial x - \sigma EB}{\sigma B^2}$$

The averaged current density can be expressed as

$$\langle J\rangle = \sigma u_0 B\langle f\rangle(1-\alpha) \tag{1}$$

where

$$\alpha = R/(R+R_i) = E/u_0 B\langle f\rangle \tag{2}$$

Other quantities of interest are

$$\langle P\rangle = \sigma u_0^2 B^2\langle f\rangle^2(1-\alpha)\alpha \tag{3}$$

$$\langle V\rangle = \mu u_0^2\langle(\partial f/\partial y)^2\rangle \tag{4}$$

$$\langle D\rangle = \sigma u_0^2 B^2\langle f\rangle^2(\langle f^2\rangle/\langle f\rangle^2 - 2\alpha + \alpha^2) \tag{5}$$

and

$$\langle W\rangle = \sigma u_0^2 B^2\langle f\rangle^2(1/\langle f\rangle - \alpha) \tag{6}$$

The simplifying assumptions implied by the above relationships are familiar and will not be repeated here. We note that $\langle P \rangle$ is a parabolic function of α, just as in the inviscid case. The maximum obtainable power, occurring at $\alpha = 0{\cdot}5$, is reduced by a factor of $\langle f \rangle^2$ from the inviscid prediction.

It is expected that $\langle f \rangle$ will be smaller than or equal to unity; $\langle f \rangle = 1$ represents the limiting case of one-dimensional inviscid flow. The non-uniform velocity profile introduces a non-zero viscous dissipation term $\langle V \rangle$. In addition, with the no-slip condition at the wall, there will be a region near the wall where the fluid velocity is smaller than the ratio of the electric field to the magnetic field ($u < -E/B$). In this region, the current is in the direction of the applied electric field rather than opposing it, and m.p.d. acceleration of the plasma prevails rather than power generation. A portion of the electric power generated in the main stream is consumed for the plasma acceleration in the boundary layer. Joule dissipation exists both in the main stream and in the boundary layer.

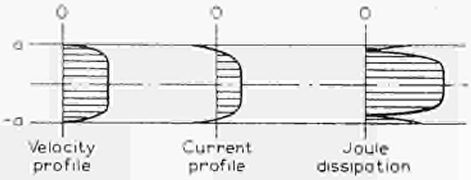

Fig. 2
Current profile and Joule dissipation

The velocity and the current profiles and the Joule dissipation will be of the shape sketched in Fig. 2. It is of interest to note that the viscous dissipation $\langle V \rangle$ is independent of α, while the Joule dissipation $\langle D \rangle$ is dependent upon α. The fact that the viscous-dissipation term may be small does not imply that the combined effects of viscous dissipation and the reversed current in the boundary layer are unimportant. In fact, it might be argued that, for a given viscosity and a given value of main-stream velocity, the average viscous dissipation is inversely proportional to the boundary-layer thickness, while the reversed-current region (and hence the total reversed current) is directly proportional to the boundary layer thickness. Thus a small viscous dissipation implies a thick boundary layer and a heavy reversed-current loss. On the other hand, while a thin boundary layer results in a small reversed current, it also implies a large velocity gradient and a large viscous dissipation. From the following analyses, it is seen that the viscous dissipation is a less serious loss than the reversed-current loss, and the boundary-layer thickness in the generator channel must be kept at a minimum, even though this means a comparatively high viscous dissipation.

On the basis of the same α and B, two ratios are formed to compare the viscous (subscript v) and inviscid (subscript i) powers:

(*a*) For the same pressure gradient $(\partial p/\partial x)_v = (\partial p/\partial x)_i$:

$$\frac{\langle P \rangle_v}{P_i} = \langle f \rangle^2 \frac{1-\alpha}{1-\alpha\langle f \rangle} \tag{7}$$

(*b*) For the same amount of work done by the fluid in pushing itself through the channel, $W_i = \langle W \rangle_v$:

$$\frac{\langle P \rangle_v}{P_i} = \langle f \rangle \frac{1-\alpha}{1-\alpha\langle f \rangle} \tag{8}$$

The ratios are functions of α and $\langle f \rangle$ only. The factor $\langle f \rangle$ is related to the dimensionless displacement thickness, ϵ, by $\langle f \rangle = (u_{max}/u_0)(1-\epsilon)$, where ϵ is usually much smaller than unity. The above ratios can be expanded in terms of ϵ. Thus, eqn. 8 becomes

$$\langle P \rangle_v/P_i = 1 - (1+m)\epsilon + (1+m)m\epsilon^2 - \ldots$$

Consequently, for $m\epsilon < 1$, smaller values of ϵ lead to better generator performances, which verifies the statement in the above paragraph. It is also clear from the above that the effects of viscosity are more pronounced at higher values of α. In addition, $m\epsilon > 1$ corresponds to $\alpha > u_0/u_{max}(1+\epsilon)$. Since this means that α is very close to unity, implying very low power densities, it is not of great interest for the present study.

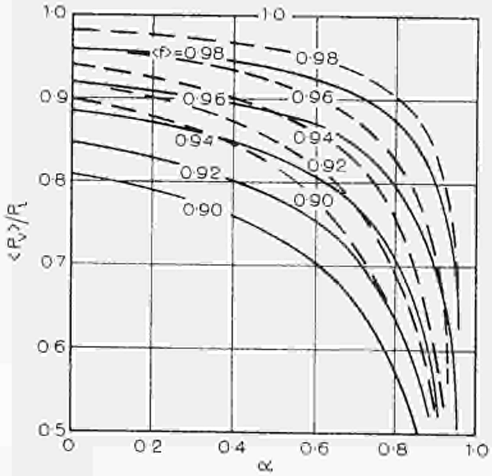

Fig. 3
Power ratios

——— $(\partial p/\partial x)_v = (\partial p/\partial x)_i$
— — — $\langle W \rangle_v = W_i$

The power ratios are plotted in Fig. 3 for values of $\langle f \rangle$ between 0·90 and 1·00. The ratios given above are related to a particular station, x_0, of the generator channel. To estimate the overall performance of the generator, they can be integrated and averaged over the channel length, l. A simpler approach is to study the parallel flow problem and then estimate the deviation from this in an actual channel. The latter approach is adopted here.

Hartmann Flow

Making use of Hartmann's solution[5] to the parallel, incompressible flow problem, one obtains

$$\langle f \rangle = 1 \tanh \beta/\beta \simeq 1 - 1/\beta$$

where the approximate expression is for large values of β (Hartmann number). For $\beta > 5$, which is true for most generators, the approximate expression is accurate to the third figure. Expressions corresponding to eqns. 3–8 can easily be derived.

Consider a possible generator design with argon, helium or air seeded with 1% potassium as flow medium and operating at a temperature of about 2500°K, a pressure of about 1 atm, a velocity of about 1000 m/s, with channel dimensions $2a = 10$ cm, $2b = 50$ cm, $l = 5$ m and with $B = 10\,000$ G. The Hartmann number, β, is approximately 15. Thus, at $\alpha = 0{\cdot}5$, the power ratios given by eqns. 7 and 8 are 88% and 82%, respectively. At $\alpha = 0{\cdot}8$, the ratios are 75% and 70% only. For a larger channel, say $2a = 20$ cm, the corresponding ratios are 93%, 90%, 85% and 82%. The channel height, $2a$, unfortunately is limited by magnet design and other considerations and cannot be arbitrarily large. The above ratios indicate that the effects of viscosity should not be ignored in actual generator designs. For experimental generators with very small channel dimensions, the effects of viscosity will be highly important. As previously mentioned a reduction of power by a factor of 4 was estimated[2] for $\epsilon = 0{\cdot}20$, although the dependence between the reduction in power and the load factor was not given. The factor of 4 is much higher than values obtainable from eqns. 7 or 8 for values of $\alpha < 0{\cdot}9$. The disagreement is apparently a result of the simplifications made in the equivalent circuit analyses[2]. Since the Hartmann number is directly proportional to $\sqrt{\sigma}$, higher σ values result in better generator performance. Thus, operating at temperatures higher than 2500°K and pressures lower than 1 atm, the use of better seeding materials or non-equilibrium effects will tend to minimize the effect of viscosity.

Boundary-Layer Flows

To estimate the deviation from Hartmann flow, consider the momentum equation for a constant σ and μ boundary layer. For the constant free-stream velocity flow, the pressure force exactly balances the Lorentz force in the free stream and the momentum equation can be written as

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 y}{\partial y^2} + \frac{\sigma(u_0 - u)B^2}{\rho} \qquad (10)$$

where the last term represents the sum of the pressure force and the Lorentz force in the boundary layer. It can be shown[7] that, for some types of generator channel, the boundary-layer thickness is of the form $\delta \simeq \delta_0(1 - a\varepsilon^{-mx})$ where δ_0 and a are constants. Since the sum of the pressure, viscous, and Lorentz forces should be approximately equal to the inertia force, one obtains

$$\delta \propto \left(\frac{\nu x}{u_0}\right)^{1/2}\left(\frac{1}{1+mx}\right)^{1/2} \qquad (11)$$

Thus, the Blasius type of boundary layer is expected for $mx \ll 1$. For large values of mx, δ is essentially independent of x, and the flow is of the Hartmann type. Fay[8] suggested that boundary layers of constant thickness are always experienced near the downstream end of the channel. For the generator characteristics previously mentioned (l = 5m, etc.) the value of ml is about 1 for argon or air and about 10 for helium. Thus, for helium, Hartmann-type flow prevails for the major portion of the channel. For argon and air, the averaged (over l) boundary-layer thickness is somewhat less than the Hartmann boundary-layer thickness. The power ratios for air and argon should therefore be higher than those previously estimated. It should be noted, however, that the boundary layer usually has a finite (non-zero) thickness at the entrance of the generator section. The deviation from Hartmann-type flow inside the channel is not expected to be large. It appears advantageous (to reduce viscous effects) to employ a heavier plasma as the flow medium since a heavier plasma results in a thinner boundary layer. This conclusion, however, is subject to consideration of the flow turbulence.

Turbulence and Compressibility

Hartmann's problem is concerned with laminar flows of incompressible media. On the basis of the experimental evidence reported by Hartmann and Lazarus[6] and by Murgatroyd[9] on the flow of mercury, it is expected that the flow in some generator channels is turbulent. The presently available data, however, do not permit definite predictions on whether a flow in a certain configuration is laminar or turbulent. Harris[10] correlated the data of References (6) and (9) and, using dimensional analyses and a semi-empirical approach, computed mean (time-averaged) velocity profiles and current distribution for the turbulent hydromagnetic channel flow. It is possible to make use of the results of Harris to estimate the power ratios for turbulent flows. These ratios are expected to be smaller than the corresponding laminar values. Since a smaller density means a smaller Reynolds number, it is possible that in some flow channels helium plasma will be stable laminar while argon or air flows will be turbulent. Thus, for such configurations, it may be advantageous to use a lighter flow medium. Also, if the compressibility of the plasma is taken into consideration, it is expected that the thickness of the boundary layer will be larger than for incompressible flow. From the previous analyses, it is seen that a thicker boundary layer is accompanied by a more severe loss, and the results obtained here therefore represent relatively optimistic estimates. Wall cooling generally results in a smaller boundary layer thickness and thus a smaller loss due to viscosity. It is, however, usually accompanied by other losses and is not expected to be an effective means of improving the overall efficiency. Other means of reducing the boundary-layer thickness, such as boundary-layer suction, may prove to be useful in improving the efficiencies of m.p.d. generators.

Effects Associated with Electrodes

The presence of electrodes, which are assumed to have negligible viscous effect in the above analyses, imposes an additional penalty on the performance of the generator. Using an approach similar to the one used above and applying Resler and Sears[11] results for flow between two parallel electrodes, an expression similar to eqn. 8 can be obtained. It is found that the reduction in power based on $W_i = W_r$ due to the electrodes is indeed much smaller than the reduction due to the insulators. The principal viscous effect of the electrodes is to produce a non-uniform electric field in the channel. The average voltage is proportional to the flow rate through the channel.

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